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A TUNED SINGLE PARAMETER FOR REPRESENTING CONJUNCTION RISK

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Satellite conjunction assessment risk analysis is a subjective enterprise that can benefit from quantitative aids and, to this end, NASA/GSFC has developed a fuzzy logic construct—called the F-value—to attempt to provide a statement of conjunction risk that amalgamates multiple indices and yields a more stable intra-event assessment. This construct has now sustained an extended tuning procedure against heuristic analyst assessment of event risk. The tuning effort has resulted in modifications to the calculation procedure and the adjustment of tuning coefficients, producing a construct with both more predictive force and a better statement of its error.

INTRODUCTION

THE identification of close approaches between two satellites in space, or conjunction assessment, is a straightforward process by which the estimated states of two closely orbiting objects are propagated over time and compared. Although there are numerous complexities to this process, such as state and state uncertainty estimate techniques, propagation methods, force modeling parameters, and physical property modeling of the objects themselves, the process is objectively governed by the underlying physics and dynamics. Conversely, conjunction assessment risk analysis (CARA) is the interpretive evaluation of the predictions resulting from that process. It comprises the evaluation of satellite conjunction events against a set of risk criteria (including the ability to detect and predict conjunctions both confidently and accurately), the probability of their occurrence, and the severity should they in fact occur. Hence, effort has been devoted to this aspect of the satellite conjunction problem over the last decade.

The first major milestone in the satellite conjunction risk assessment field was introducing the probability of collision (P_c) calculation.[1,2] The P_c is a metric of the likelihood of the predicted close approach between two orbiting objects resulting in a collision between the objects' hard-bodies. It considers the size of the objects involved and the geometry and predicted proximity of the close approach, as well as the uncertainty in the position of the objects as described by their estimated position covariances. Given precise and reliable inputs to this calculation, the P_c should

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be the true assessment of conjunction risk. However, these inputs are never precisely known and come with their own, usually poorly characterized, uncertainties. If these inputs are inaccurate or unrepresentative, the result is an unrealistic assessment of the risk level and a potential unnecessary and risky conjunction avoidance maneuver. The state estimate covariance of the secondary object is all too often an example of this inaccurate input. The orbit determination (OD) process can produce unrealistic covariances due to uncorrected biases in the input observations, correlation among observations, and shortcomings in the force models. If one is convinced that unrepresentativeness of the data that feeds the P_c calculation is a significant problem, then use of additional risk assessment parameters is necessary in order to more properly evaluate the risk level of a satellite conjunction event.

In addition to the analysis of satellite conjunction risk, communicating that risk to the satellite Owner/Operator (O/O), who may have the ability to perform mitigative action against those risks, is of perhaps equal importance. As previously mentioned, the P_c can be a statement of the conjunction risk, but its value typically fluctuates, often substantially, over the course of a conjunction event, which makes it difficult to use as a parameter by which to communicate risk to an O/O, especially if the O/O is not familiar with the subtleties of conjunction assessment. Operational experience at NASA Goddard Space Flight Center (GSFC) has shown that NASA O/Os desire a less ambiguous and more quantitative statement of the conjunction risk analysis, whether it be for reporting and historical purposes or for determining precautionary actions.

In an attempt to find a comprehensive conjunction risk analysis metric, Frigm (2009) proposed a fuzzy logic construct – called the F-value – with the goal of capturing and conveying the conjunction risk analysis in a single numerical value.[3] The F-value is a rather flexible construct that can be used with a diverse set of O/Os to communicate conjunction risk. However, thus far this construct has essentially been based on heuristics, without a rigorous tuning and verification approach. The analysis presented in this paper is an attempt to tune the F-value using empirical data, as well as to more precisely determine its predictive and communicative power.

F-VALUE REVIEWED

Equation (1) presents the definition of the F-value, where *m* and *n* are the total number of risk and quality assessment parameters, respectively.

$$F_{value} = \left(\frac{1}{\sum_{i=1}^{m} a_i} \sum_{i=1}^{m} a_i f_i \right) \cdot \left(\frac{1}{\sum_{j=1}^{n} b_j} \sum_{j=1}^{n} b_j f_j \right) \tag{1}$$

The full details of the development of the F-value construct are given in Frigm (2009), but a summary is provided here. Each contribution to the overall F-value is made by means of a *membership function*, which is a relationship that maps the value of a certain parameter into a pre-established, standardized numerical scale, typically a unitless range of 0-1 or 0-10. Each $f_{i/j}$ in Equation (1) above represents one of these mappings for a contributing parameter, such as probability of collision or tracking density, whereas each $a_{i/j}$ represents the associated weighting coefficient. This approach allows parameters of different base units to be combined in a single expression. In the construct shown in Equation (1), the first set of terms on the right-hand side reflects the conjunction's essence by including two parameters: the smallest miss distance (among all three RIC[*] miss components) and the P_c, each mapped into a 0 to 10 value, with a higher value indicating greater severity. The second set of terms on the right-hand side derive from an attempt to characterize the quality of the OD feeding the assessment by including three parameters: the logarithm of the determinant of the combined covariance, a quantification of tracking density, and the time interval between the last sensor metric observation of the secondary object and the creation time of the orbital conjunction message (OCM); each of these parameters is mapped via a membership function to a value from 0 to 1.

The total F-value construct is thus the product of a weighted average of the conjunction severity terms (first term) and the OD quality terms (second term). The quality terms (bounded between 0 and 1) can essentially be seen as a scaling factor applied to the actual severity terms (bounded between 0 and 10). The resulting F-value thus takes on a value between 0 and 10, with a higher value indicating higher risk.

The initial deployment of the F-value established heuristic membership functions and set the weighting coefficients to unity, exploring the power of the construct with these provisional settings. The present effort seeks a more rigorous construct by reviewing and modifying the membership function relationships and determining values of the weighting coefficients in order to maximize the construct's predictive force.

LEVELS OF CONJUNCTION EVENT SEVERITY: THE WORK TIER

In order to tune the F-value effectively, its output needs to be compared to some external measure of event severity. This measure is a subjective assessment by the operational analysts that consistently perform this function on a daily basis. It was found that a reasonable measure of event severity is the amount of "work" that each event generates. Different levels of severity necessitate different types and amounts of analytical products to be generated, so an examination of which particular products were produced for any given event is a good indication of the event's seriousness and therefore operational risk. To allow workload rankings to be assigned, five work-product-based groups, called *work tiers*, were developed. Table 1 provides a description of each of these work tier levels. Work tier 4 (execution of collision avoidance maneuver) is not an analytical product *per se*, but because it represents the strongest possible defensive action that can be taken, it seemed appropriate to make it the capstone of a set of levels intended to reflect event severity. Given the retained event history in the CARA operational database, it is straightforward to assign a given work tier level to any given conjunction event. The work tier level can thus be used as an indication of event severity/risk that the F-value (or any similar risk assessment construct) can attempt to predict.

[*] Radial, In-track, Cross-track

Table 1: Division of Work Tier Levels

Work Tier Level	Description
0	The CARA* team provides the Owner/Operator with a Summary Report via automated e-mail listing all satellite conjunction events within a specified screening volume, including trend plots.
1	The CARA team notifies Owner/Operator via e-mail or phone providing the CARA team's assessment of the satellite conjunction event's risk. The CARA team analyzes four main figures of merit: conjunction geometry, uncertainty analysis, conjunction evolution and trends, and orbit determination quality.
2	The CARA team provides a formal briefing to the Owner/Operator. The presentation provides additional information to describe the significant attributes of the satellite conjunction event leading to the assessment and associated risk. These include output from the CARA team's in-house tools: trend plots, visualizations, sensitivity analyses, conjunction geometry, maneuver options, etc.
3	The CARA team provides a recommendation from which the Owner/Operator decides to begin the maneuver planning process. The CARA team provides assistance in determining possible maneuver options that would mitigate any risk associated with the conjunction. The CARA team also analyzes/assesses predicted post-maneuver conjunction events and their associated risks.
4	The Owner/Operator weighs the conjunction risk (from the assessment and recommendation of the CARA team) and all other mission risks and decides to perform a Risk Mitigation Maneuver (RMM). A debrief is typically provided.

Experiences of the CARA team show that most O/Os prefer to receive a notification when a satellite conjunction event is within a certain tolerance threshold, regardless of whether the event is expected to become significant. This threshold, called a screening volume, is typically defined as a rectangular box centered on the primary spacecraft, and any secondary space object violating the parameters of this volume triggers a notification of a potential satellite conjunction event.[4] Regardless of the risk severity for a particular satellite conjunction event, the notification of a screening volume violation is sent to the O/Os via automated e-mail. At this point, the work tier level of zero is assigned to the event. If the conjunction event is eventually elevated to a higher work tier level, it is only then treated as a significant event (work tiers 1-4); all work tier level 0 events are considered insignificant.

F-VALUE TUNING DATASET

The F-value tuning process utilized 4627 unique satellite conjunction events taking place from July 2010 to May 2011. There were 151 events with work tier level 1 or higher, while the remaining events were assigned a work tier level of 0. Each satellite conjunction event contained anywhere from one to fifteen Orbital Conjunction Messages (OCM). An OCM is a data product that is delivered by the Joint Space Operations Center (JSpOC) providing useful information about a satellite conjunction event, including the time of closest approach (TCA), the RIC miss distances,

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the covariance matrices, and other amplifying information. Typically, the first notification of a satellite conjunction event is provided approximately seven days in advance of the actual TCA. Subsequent OCM updates are normally delivered at a once to twice per day frequency depending in part on the availability of the new tracking data. Accompanying information with each OCM allows the calculation of the F-value. Apart from the computation of the F-value, each event's work tier level assignment is also known. All of these data are preserved in the CARA operational database and were thus easily mined for the present analysis.

F-VALUE TUNING PART I: MEMBERSHIP FUNCTIONS

The F-value construct as presently deployed comprises four risk and three quality assessment parameters for which membership functions are needed, although the number of the actual risk parameters used is reduced to two from four through a supremum test. These seven parameters are: radial, in-track, and cross-track separations at the TCA; probability of collision; tracking density (tracks used per day); time since last observation (days); and the "size" of the combined covariance. The first four constitute the category of risk assessment parameters, while the last three are quality assessment parameters. The supremum test chooses the separation parameter (of the three RIC components) whose membership function evaluation indicates the highest risk.

For several years, routine conjunction risk assessment has been performed for NASA missions in a variety of orbit regimes, including Low Earth Orbits (LEO), Geosynchronous Orbits (GEO), and Highly Elliptical Orbits (HEO). When it comes to physical RIC separations, the risk level in each of these orbit types is assessed differently – certain separation thresholds may be alarming in GEO while quite acceptable in LEO. In order to avoid defining a separate set of membership functions for each orbit regime, these separation parameters were normalized as a percentage of the maximum separation value for the given orbit regime. The maximum RIC separations commonly used for screenings are 0.5 km, 5 km, and 5 km, respectively.[4]

The preliminary forms of membership functions were already presented in Frigm (2009).[3] This initial concept was based on techniques utilizing the intuition and direct operational experience of the author. Since that time, formal consultations with the entire CARA team have enhanced and evolved the proposed membership functions. In addition to establishing the form, careful consideration of the membership function end-points was necessary, as these points establish thresholds for which the risk essentially cannot be increased (even though in some cases the associated parameter value could be) and at which the parameter essentially poses no risk at all. The present membership function definitions are given below in Figures 1-7.

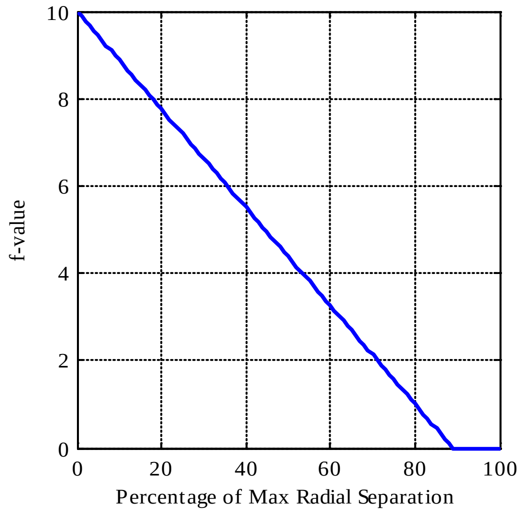


Figure 1: Radial Separation Membership Function

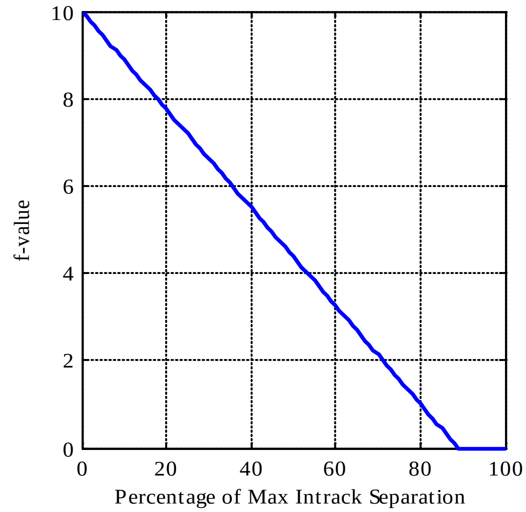


Figure 2: In-track Separation Membership Function

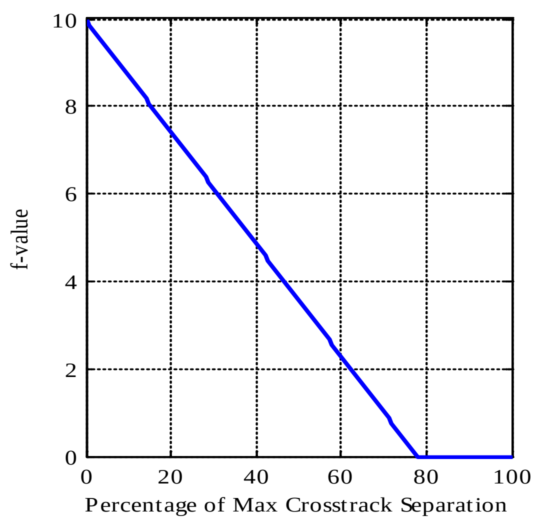


Figure 3: Cross-track Separation Membership Function

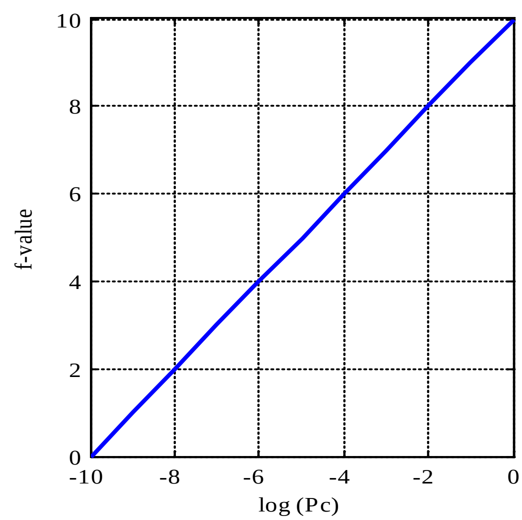
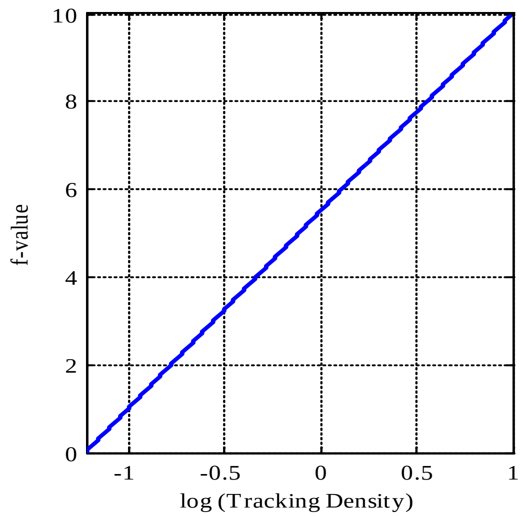


Figure 4: Probability of Collision Membership Function

The interpretation of Figures 1-7 and their use are straightforward. An OCM for a particular satellite conjunction event will contain all the necessary information to calculate the domain value for each contributing parameter. For example in Figures 1-3, RIC components of the miss distance are directly provided on the OCM. The P_c value is also now being provided on the OCM, but that particular value was not utilized in this study. An in-house developed P_c calculation algorithm was implemented. Given the dynamic range of the $P_{c,}$ the use of a logarithmic scale in this scenario is appropriate.



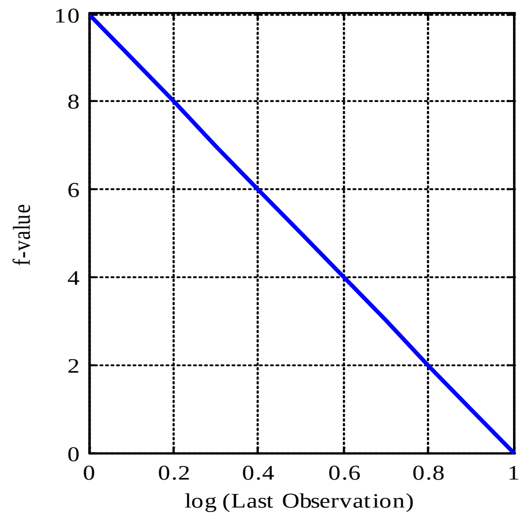


Figure 5: Tracking Density Membership Function (Tracks Used/Day)

Figure 6:Time Since Last Observation Membership Function (Days)

In Figures 5-6, the tracking density and the time since last observation are computed from amplifying information provided within the OCM. In particular, tracking density is defined to be the number of tracks used per differential correction span. The time of the last observation given in days is a difference between the creation time of that OCM and the time of the last metric observation of the secondary object.

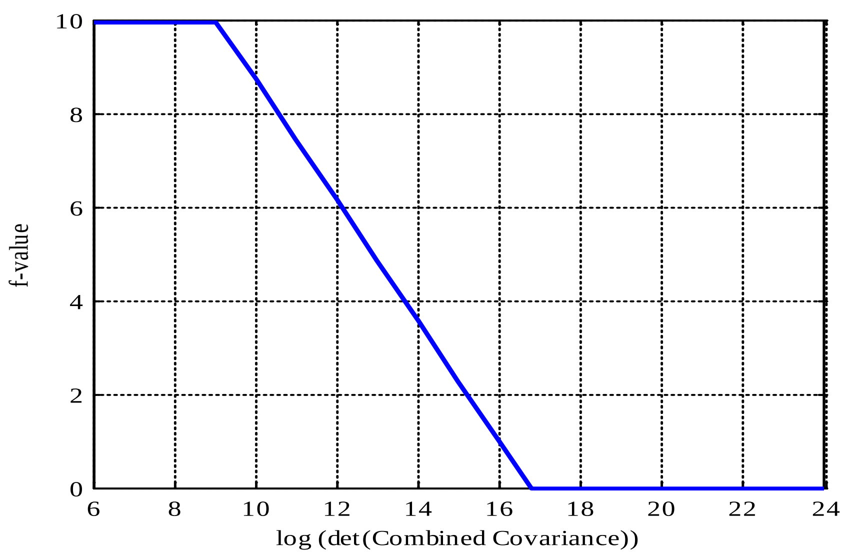


Figure 7: Combined Covariance Membership Function

The membership functions for four of the F-value's input parameters deploy a logarithmic transformation of the data, which allows for the overall simpler representation as a linear relationship. It bears mention that combining of the primary and secondary object's covariances in Figure

7 is performed in a common inertial reference frame (J2000). The determinant of the combined covariance is useful because it gives a single-value representation of the "size" of the combined uncertainty ellipsoid. A determinant of a matrix is a product of its eigenvalues, which for a nearly-diagonal covariance matrix is essentially the product of the uncertainty variances.

The membership functions allow the calculation of all of the "raw" f-values (f_i and f_j) in Equation (1) for any given event. When tuning coefficients are established, an overall F-value can then be calculated; and the F-value, in concert with an established F-value threshold, can be used to determine whether an event is significant or not (work tier level greater than 0) or, potentially, the expected final work tier level.

F-VALUE TUNING PART II: CANDIDATE TUNING COEFFICIENTS

A fully defined F-value construct will have a single set of weighting coefficients, one for each membership parameter, and four different F-value thresholds, one for each significant work tier level. The calculated F-value for an event would indicate that the event belongs to a certain work tier if the F-value exceeds the designated threshold for that work tier. Obviously, an event with the F-value not exceeding the threshold of the work tier level 1 is labeled as an insignificant event. It is noted here that for this analysis, a representative F-value for an event is the largest F-value that is produced throughout the evolution of that event.

From this formulation, two types of errors are possible when the F-value prediction fails to correctly determine the work tier level of a satellite conjunction event. Table 2 provides their respective definitions.

Table 2: Type 1 and Type 2 Error Definitions

Error	Description
Type 1	This type of error is flagging a satellite conjunction event as a higher tier level (*i.e.*, tripping the threshold for the work tier level under investigation) when in fact it is a lower tier level. While this error type is undesirable, its occurrence is generally preferable to a type-2 error.
Type 2	This type of error is flagging a satellite conjunction event as a lower tier level (*i.e.*, failing to trip the threshold for the work tier level under investigation) when in fact it is a higher tier level.

It is evident that the cost function to be minimized for the optimization problem of finding the optimal set of F-value coefficients is a binary one, in that for any given event the F-value construct either predicts a certain work tier correctly or it does not. Such cost functions often fare poorly with classical constrained optimization approaches in that the process usually does not produce a global minimum. In other words, a small change in initial conditions results in the convergence of an iterative method to a different local minimum. Also, such minima may be identical yet be reached with substantially different coefficient sets. As an alternative method, the number of tuning coefficient combinations is small enough that a *brute force* approach is possible. All of the tuning coefficients are systematically varied in a nested manner from 0 to 1 in the increments of 0.1, and the candidate F-value thresholds are systematically varied from 0 to 10 also in the increments of 0.1. A single iteration of the brute force algorithm selects a specific combination of tuning coefficients and an F-value threshold, and makes a binary prediction for a particular work tier, for all 4627 conjunction events, after which the overall count of type-1 and type-2 errors for that coefficient and threshold set is determined. This process is repeated until all designated coefficient combinations and F-value thresholds have been exhausted. An example of the results for a particular run through the brute force method for tier 1 is presented below in Figure 8.

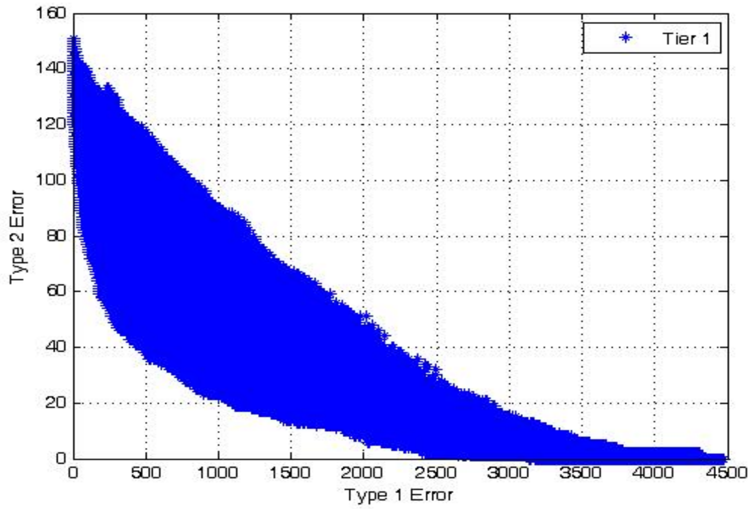


Figure 8: Work Tier 1 Prediction Region

Ideally, if the F-value's predictive capabilities were flawless, then the count for both types of error would be zero for the set of events analyzed. However, in virtually all real applications, there is a trade-off between the two error types, meaning that one can improve the outcome for one error type at the cost of worse performance for the other error type.

It is evident that the lower frontier of the shaded region in Figure 8 gives the preferable trade-off between the two error types. For example, point (1100, 20), which is on the lower edge, is preferable to (2700, 20), because if one is to endure 20 type-2 errors, it is better that they come accompanied with only 1100 type-1 errors rather than 2700 type-1 errors. A curve that provides only the desirable edge of a trade-space such as that in Figure 8 is called a *sufficient frontier*.

A sufficient frontier plot is a useful means by which one can choose the desired trade-off ratio between the error types. However, it must be emphasized that there is no "correct" type-1 / type-2 error ratio, except in those rare instances in which a 0/0 ratio happens to be possible. The question then is of the number of additional type-1 errors that a user is willing to endure in order to reduce the type-2 error by a certain amount.

Figure 9 below presents the full outcome of the brute force iterative method and gives four such sufficient frontier curves for each respective tier. It is helpful to remember that such curves do not report the results for all 4627 events because they give only the sufficient frontier; each eliminated point is not represented because some other point, which is shown, is necessarily a superior choice.

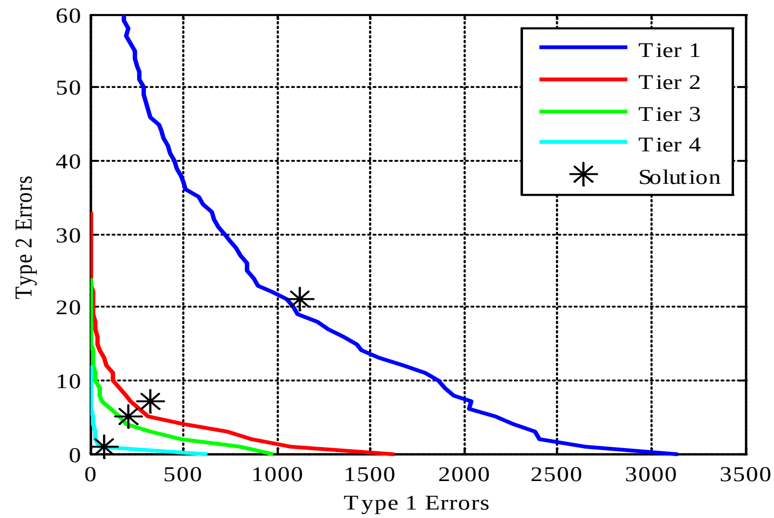


Figure 9: Work Tier Predictions

The optimal set of tuning coefficients chosen is the one that results in a favorable placement along the sufficient frontier curves. The choice of an *optimum* point is of course subjective and depends on one's risk tolerance and the degree of encumbrance that type-1 errors will present. The F-value tuning coefficients and thresholds that were ultimately chosen are summarized in Table 3.

Table 3: Optimal F-value Tuning Coefficients and Thresholds

F-value tuning parameters					F-value Threshold			
Risk Parameters		Quality Parameters			Tier Level 1 – 4			
RIC Miss Distance	P_c	Combined Covariance	Tracking Density	Time Since Last Observation	Tier 1	Tier 2	Tier 3	Tier 4
0.0	1.0	0.6	0.1	0.0	1.0	2.5	2.7	3.7

The performance of the values in Table 3 is shown in Figure 9 by the placement of asterisks along the sufficient frontier curves (labeled as "solution" asterisks in the figure legend). The desire was to select a single set of tuning coefficients and four F-value thresholds that would yield good performance for prediction of each of the four work tiers. To do this, optimal performance for each of the work tier predictions was compromised slightly; this is why the solution asterisks do not lie precisely on the sufficient frontier but slightly behind it.

The actual rate of significant events detected versus false-alarms is summarized in the Table 4 below. The large number of type-1 and type-2 errors encountered, even for the selected optimum points, is somewhat dispiriting, but it must be remembered that the F-value is trying to predict a process that is heavily subjective.

Table 4: Performance of the Optimal Set of F-value Tuning Coefficients

	Significant Events Detected	False – Alarm Rate
Tier Level 1	130/151 (~ 7 out of 8)	1116/4627 (~ 1 out of 4)
Tier Level 2	26/33 (~ 4 out of 5)	322/1631 (~ 1 out of 5)
Tier Level 3	19/24 (~ 4 out of 5)	203/976 (~ 1out of 5)
Tier Level 4	11/12 (~ 9 out of 10)	79/633 (~ 1 out of 8)

COMPETING CONSTRUCT: PROBABILITY OF COLLISION

Some analysts maintain that P_c alone is an adequate quantifiable metric of satellite conjunction risk level, and this is not an unreasonable position, for in principle all the relevant data (miss distance, object size, and OD uncertainty as represented by the covariance) are included in the calculation. So it is a natural question whether using the P_c alone, in the same manner as the F-value, can produce a competitive result. The same iterative brute force process as previously discussed was used, but the largest calculated P_c value throughout the evolution of an event, rather than the calculated F-value, was chosen to be the representative datum for each event. No tuning coefficients were thus necessary. The P_c candidate thresholds chosen for the process ranged from 10^{-10} to 0.9 in logarithmically-determined size increments. The optimal set of P_c threshold values emerging from the analysis is given below in Table 4 and the associated sufficient frontier plot in Figure 10.

Table 5: Optimal P_c Threshold Values

P_c Threshold			
Tier 1	Tier 2	Tier 3	Tier 4
$2.0 * 10^{-6}$	$1.0 * 10^{-3}$	$1.5 * 10^{-3}$	$2.0 * 10^{-3}$

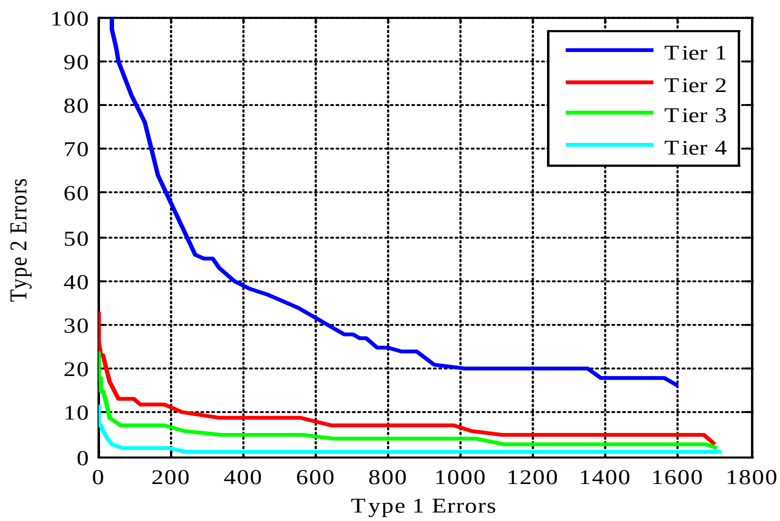


Figure 10: Work Tier Predictions Using Only P_c Data

Here, the performance using only the P_c data is very similar to that of the F-value, with some minor differences. The P_c approach slightly outperforms the F-value in predicting work tier level 1 (or higher) but underperforms the F-value for prediction of work tiers 2, 3, and 4 (or higher). It is difficult to make a definitive choice between the two on these results alone, so one should examine the additional consideration of event stability (next section) before making any final recommendations.

F-VALUE TUNING PART III: EVALUATION STABILITY

Each unique event on average produces several OCM updates, and what is desired is that as large a number of these OCMs as possible produce the same correct evaluation (in terms of predicted work tier) by the evaluating factor, whether it be F-value or $P_{c.}$ As an illustrative example, consider an event that produced several OCMs; for each of these OCMs, there is an associated F-value and P_c that have been calculated. Each work tier level has its respective F-value and P_c thresholds found in Tables 3 and 5; the F-value or P_c evaluations that exceed each threshold would indicate that the event belongs to that work tier. The stability evaluation process iteratively goes through each unique satellite conjunction event for all four different significant work tiers and flags a particular OCM update within an event as a success if the calculated F-value or P_c matched the assigned work tier level for that event; otherwise it is flagged as a mismatch. If all of the OCMs for an event produce the correct work tier level, then the *stability index* for that event would be 100%; if only 75% of the OCMs produced a prediction that aligned with the actual work tier level, the stability index would be 75%, &c. Of course, percentages closer to 100%, on a 0-100 scale, indicate more stable correct evaluations over event history.

For this type of data result, it is most informative to display the result as a cumulative distribution function (CDF) plot. Figure 11 shows the stability performance comparison between F-value and $P_{c.}$ Reading the graph is not entirely straightforward, so a more detailed explanation is provided below. The graphs have been reworked somewhat to put them in a more standardized CDF form.

Beginning with the upper left plot, the CDF values that lie on the y-axis give the number of events characterized with perfect stability (100% agreement among OCMs for a single event). As can be seen, the F-value outperforms the P_c slightly at this point (80% to 77%), and it maintains this edge at the 50% stability (x-axis) point as well (88% to 84%). For work tier levels 2 and 3 (upper right and lower left), the situation is inverted in that the P_c outperforms the F-value slightly at the y-axis (100% stability situation); but by the 50% level both lines have converged, and there are essentially no events worse than this level anyway. For work tier level 4, there are essentially no non-100%-stable cases for either parameter.

While the F-value fares marginally better in some circumstances and the P_c in others, both approaches render essentially the same performance. Perhaps one could say that the F-value's better performance *vis-à-vis* stability (which is in the work tier level 1 case) is more significant because the separation between work tier level 0 (a non-significant event) and greater than work tier level 0 (a significant event) is the most useful distinction to make at the point at which the event is initially detected (usually seven days from TCA), so a more stable performance is desirable in this situation. But even if one grants this, the improved performance cannot be said to be significant.

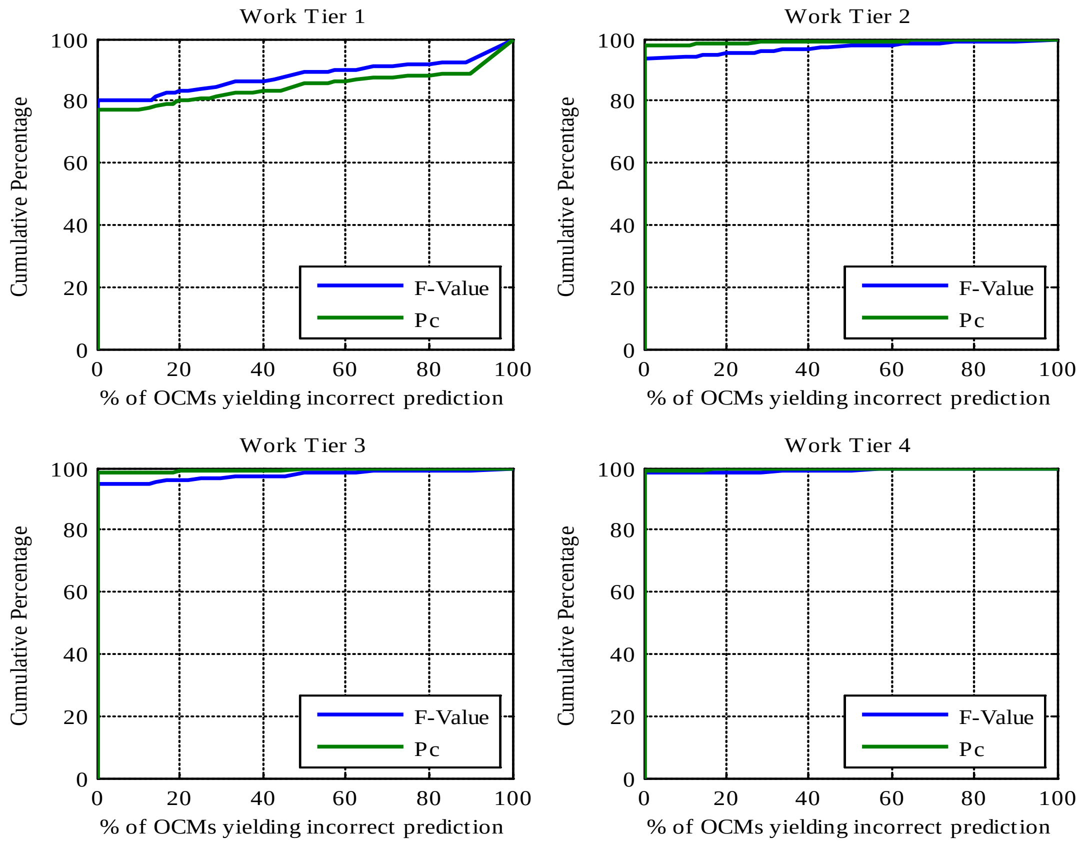


Figure 11: Work Tiers 1-4 Prediction Uniformity

CONCLUSION: DISCUSSION AND FUTURE WORK

Conjunction assessment risk analysis is the process of determining if a predicted close approach between two orbiting objects poses a risk of collision to those objects. The probability of collision computation emerged as a metric for determining the likelihood of a conjunction resulting in a collision. It has been observed operationally at NASA / GSFC that the accuracy of P_c as a risk metric is subject to necessary understanding of the underlying data used in the computation. Conjunction risk analysis, therefore, is this process of qualitatively and quantitatively assessing this close approach data.

One construct employed at NASA / GSFC for quantifying this conjunction risk, including the qualitative and quantitative assessment of the close approach data, is the F-value. The F-value, in its present form, is a heuristic model that uses traditional close approach data, such as miss distance and $P_{c,}$ but also the required understanding of conjunction metadata, such as tracking data density, to more thoroughly quantify conjunction risk. The analysis presented here is an attempt to tune that heuristic model with operational analyst data. While the analysis does show success in the F-value discriminating conjunction events between those warranting further analysis and non-risk conjunctions events, it also shows that the current construct may need re-visited.

Ideally, the F-value would be tuned to be able to discriminate between each of the work tiers. The inability of the tuning presented here to meet this goal is likely a result of the limited data set used to tune the model. Of the 4627 conjunctions used in this analysis, only 151, or less than 5%, of these events were in work tier level 1 or higher. Since the majority of the data resided in work tier level 0, the tuning may in fact be biased to that data set. This lack of high work tier events is a result of using the operational data set since high work tier events do not make up the majority of identified conjunctions.

Another potential concern with the tuning data available is attempting to gauge the risk associated with the conjunction by the amount of work used to analyze the conjunction via the work tier. As stated, the work tiers approach was adopted because no truth measurement of risk is available from the operational data set. However, the work tiers represent the amount of work that went into a conjunction and assume this correlates to high risk. While this correlation, in general, is true in operations, exceptions do exist. For example, a satellite with no propulsive capability could observe a very high risk conjunction; however, since there is no capability to mitigate the conjunction, the work tier would never get past a level two, where the P_c or the F-value would capture that true, high conjunction risk. The data set was not analyzed for the frequency of this type of events.

While the analysis presented here does not show that the F-value can be tuned to discriminate between all work tiers, the F-value as presented in the original paper[3] has demonstrated success in predicting and communicating stable conjunction risk in CARA operations at NASA / GSFC. Moreover, for many conjunctions, analysis experience continues to show that for the P_c alone to do an adequate job at summarizing conjunction risk, the need to examine the quality and confidence of the solutions used in the close approach prediction process cannot be overlooked. Therefore, the F-value concept as a risk metric is still believed valid and useful.

Although this tuning methodology did not provide the ability to discriminate between high work tier events, other tuning methodologies are under investigation. First, whether the tuning is being biased by the low percentage of high work tier events needs to be addressed. The current analysis can be repeated with a smaller data set that evenly distributes the number of events in each work tier. If the operational data set is not sufficient, simulated conjunction data may need to be produced and used in the tuning effort. Although simulating conjunction data may be more difficult, a truth risk metric (*e.g.*, true miss distance) can be determined for each simulated event. Tuning the F-value to the simulated true risk, versus the work tier proxy for risk, would be an added benefit. The F-value also makes an assumption of linear membership functions which were developed to match empirical experience, and constant weighting determined through the tuning process. It is suggested that perhaps the linear mapping and weighting are not sufficient. Neural networks are frequently used to determine patters from data sets and these tools may outperform the current techniques in associating conjunction data to risk.

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